national accelerator laboratory

NAL-Conf-73/61-THY
August 1973

Quarks, Partons, Triality, Exotics and Colored Glue [†]

HARRY J. LIPKIN [*]
National Accelerator Laboratory, Batavia, Illinois 60510
and
Argonne National Laboratory, Argonne, Illinois 60439

Topical Conference on Deep Inelastic Scattering
Stanford Linear Accelerator Center
July 16-18, 1973

[†] Work performed under the auspices of the U.S. Atomic Energy Commission.

[*] On leave from Weizmann Institute of Science, Rehovot, Israel.

TABLE OF CONTENTS

QUARKS, PARTONS, TRIALITY, EXOTICS AND COLORED GLUE

Harry J. Lipkin

I. INTRODUCTION

1.1 Who Needs Quarks?

The quark was invented by Gell-Mann and Zweig to explain certain regularities found in the hadron spectrum and the interactions of hadrons. The spectrum of the observed hadron states was found to consist of multiplets which were described simply in the SU(3) symmetry scheme with indications for possible other higher symmetries such as SU(6) or SU(3) $\times$ SU(3). The quark model gave a very simple explanation for the underlying basis of the multiplet structure and symmetry schemes, but left many important questions unanswered, the principal one being why quarks have not yet been observed. For a number of years now the quark model has provided a very good description of many aspects of hadron structure. It continues to be more and more successful as new data are accumulated. However, the fundamental question of why quarks behave this way and yet elude all attempts to find them remains unanswered. There is no argument against saying that the quarks have a very high mass and therefore have not yet been found and that the interactions between them are so strong that they produce bound states whose masses are very much smaller than the quark rest mass. However, it is not satisfying to explain phenomena which we do not understand by postulating the existence of a model which cannot be checked directly by experiment even though this model may ultimately turn out to be correct.

A new impetus has been given to the quark model by the experimental discovery of scaling in deep inelastic lepton scattering. The fact that scattering cross sections in the so-called deep inelastic region depend only on dimensionaless combinations of kinematic variables and not on any parameter with the dimensions of mass which sets the scale has suggested the parton model in which hadrons are composed of point-like constituents that have no scale. It is tempting to identify these constituents of the parton model with quarks, but it soon becomes clear that they are not exactly the same. Today it is fashionable to talk about "constituent quarks" and "current quarks,"[1—3] and to avoid any commitment to either as real physical particles which may be discovered some day.

There is no difficulty in principle in postulating that hadrons are made of particles that have not yet been discovered. There is a difficulty in practice; namely keeping theorists honest. As long as quarks have not been observed, the theorists are free to endow them with any properties they wish without fear of being contradicted by experiment. But freedom to choose quark properties means effectively that any quark model has a large number of adjustable parameters. With enough adjustable parameters one can fit everything and predict nothing. So in this game you have to be careful and think about what you are doing. There is no substitute for using your head.

1.2 The Nuclear Physics Approach

We shall try to stay honest by using the approach of nuclear physics.[4] The nuclear physicists consider the nucleus ^{17}O as consisting of a valence neutron and a closed shell ^{16}O core. Electromagnetic transitions between low-lying states of ^{17}O are described in terms of transitions of the valence neutron between different orbits. From the experimental transition probabilities the properties of the valence neutron are calculated, and the neutron is found to have an "effective charge" between 0 and 1. Nobody believes that this is the charge of the free neutron. Clearly a complete description of the transition involves the ^{16}O core as well as the valence neutron. But the fact that these 17 particles "conspire" to make transitions behave as if they were produced by a single nucleon having an effective charge is a useful way to parametrize the data and give physical insight. A more fundamental treatment is then given with an intuition borrowing concepts from field theory and quantum electrodynamics. The ^{16}O core is analogous to the vacuum, which can be "polarized" by the external particle to "renormalize" its charge. A useful concept is that of a "quasi-particle"[5] to describe excitations of complicated systems which appear to be single particle excitations, whereas in reality they are something much more complicated.

By analogy with valence nucleons in nuclei which have effective charges, interactions, form factors, scattering amplitudes, etc. which are very different from those of free nucleons, we can think of valence quarks in hadrons whose properties are very different from those of free quarks (if they exist). Many properties of the low-lying hadron spectrum can be interpreted in terms of these fictitious valence quarks, whose properties are determined to fit hadron data, and have no relation to properties of "real quarks." We can also think of free quarks, which may or may not be observable in nature, but which determine in some way the couplings of the electromagnetic and weak currents to hadrons. These free quarks can be used in current algebra, light cone algebra or parton model treatments of electromagnetic and weak processes under the assumption that the couplings of the currents to these quarks is well defined. These are the "current quarks." The wave function of the hadron in terms of these quarks is unknown and very complicated, like the exact nuclear wave function in terms of real nucleons. In the valence or "constituent" quark description, the hadron wave functions are simple and well known, but the nature of the quarks and their couplings, form factors, etc. are unknown phenomenological parameters which must be determined from the data. Their fundamental significance is very unclear, like that of the effective charge of the valence neutron in ^{17}O.

1.3 The Slow-Motion-Strong-Binding Limit

In our consideration of the old applications of the valence or constituent quark approach we use the slow-motion-strong-binding model of

Amnon Katz[6] as a guide to the intuition. This model considers a two-body system in relativistic classical mechanics and shows that it is perfectly reasonable for it to be bound with a binding energy nearly equal to the rest mass of its constituents. The internal motion of the particles is described by nonrelativistic equations. The center-of-mass motion (and also the interaction with a weak gravitational field) is described by a mass parameter which is just what is expected from naive considerations, the difference between the sum of the rest masses of the constituents and the binding energy. This mass can be as low as one pleases without causing difficulties. However, the nonrelativistic internal motion is described by a mass parameter which depends upon the interactions. It is no longer the simple "reduced mass" of nonrelativistic two-body problems and can be anything from the order of the mass of the constituents to the mass of the bound state. Thus measurements on the bound system only can tell us this "effective reduced mass," and give no information on the mass of the free particle.

Similar conclusions are obtained from a quantum-mechanical treatment using the Dirac equation for a single particle in an external potential.[2] The relativistic two-body problem with strong-binding is outside the competence of theoretical physics at present. But since the slow-motion-strong-binding limit exists and has the same intuitively attractive features both in the classical relativistic two-body problem and in the quantum relativistic one-body problem with external fields, it seems reasonable to use it as a basis for intuition in our nuclear physics approach to valence quarks.

1.4 Does the Quark Model Really Predict the Hadron Spectrum?

In this talk we consider the unanswered puzzles posed by one of the outstanding "successes" of the quark model, the prediction of the hadron spectrum. The empirical rule that all observed hadron bound states and resonances have the quantum numbers found in the three-quark and quark-antiquark systems is in remarkable agreement with experiment. Since no alternative explanation or description has been given for this striking regularity in the hadron spectrum, this rule may constitute evidence for taking quarks seriously. However, there has been no success in giving any explanation for this empirical rule. Why only 3q and $q\bar{q}$? Why not other configurations? I shall present an answer to this question in the framework of the simple minded nuclear physics approach to quarks. What exactly this answer means is not clear. But so far it provides the only answer to this question that has been proposed. That the question has an answer at all in any framework is interesting in itself, particularly since so far there are no other answers.

The quark model also predicts the energy level spectrum of the states constructed from the three-quark and quark-antiquark systems and observed experimentally as hadron resonances. These predictions also seem to be in reasonable agreement with experiment, but pose additional questions. To obtain agreement with the observed baryon spectrum, the symmetric quark model[7] must be used, which restricts the allowed states of the three-quark system to those being totally symmetric under permutations in the known degrees of freedom rather than totally antisymmetric, as one expects for fermions. This implicitly assumes that quarks obey peculiar statistics, or that there is a hidden degree of freedom sometimes called "color." The low-lying meson spectrum shows all the states "predicted by the quark model" without any supplementary conditions. All the states of the quark-antiquark system appear as meson resonances; there are no predicted states which are conspicuously absent.

In this talk we shall question the apparent success of the predictions of the spectrum. There is an inconsistency between the observation of bound states in all channels for $q\bar{q}$ scattering and the absence of bound states with quantum numbers of $2q\bar{q}$ and $3q\bar{q}$. If the quark-antiquark interaction is attractive in all possible channels, as indicated by the presence of bound states, an antiquark should be attracted by any composite state containing only quarks, like a diquark or a baryon, to make a bound state with quantum numbers which have not been observed. We suggest in this work that the only simple way to avoid this difficulty is to discard an apparent success of the quark model and to use the colored quark models which predict states for the $q\bar{q}$ system that are not observed as resonances.

In our discussion, we assume that quarks are very heavy, and we consider only effects on the mass scale of the quark mass. All observed particles have zero mass on this scale. The observed hadron spectrum is a "fine structure" which we are unable to resolve in this approximation. This is a reasonable approach, since as long as we are not treating spin in detail, we are unable to distinguish between a pion and a ρ meson, and are neglecting mass splittings of the order of the ρ-π mass difference. We therefore are only able to discuss whether a particle has "zero mass" and appears as an observed hadron, or whether it has a mass of the order of the quark mass and should not have been observed.

1.5 The Three Puzzles of the Quark Model

The question why only $3q$ and $q\bar{q}$ can be stated more precisely in terms of the following three puzzles:

1. The triality puzzle: With attractive interactions between quarks and antiquarks, why are three quarks and an antiquark not bound more strongly than a baryon or two quarks and an antiquark bound more strongly than a meson? Note that we are not asking about four quarks vs. three quarks. Symmetry restrictions, such as the Pauli principle with colored quarks can prevent the construction of a four quark state which is totally symmetric in space, spin and unitary spin. But there is no Pauli principle which prevents an antiquark from being added to a system of three quarks in all possible states. Thus if each quark in the baryon attracts the antiquark, some additional mechanism must be found to prevent it from being bound to the quark system.

2. The exotics puzzle: Even assuming some mysterious symmetry principle which prevents fractionally charged states from being seen, why are there no strongly bound states of zero triality, like those of two quarks and two antiquarks or four quarks and one antiquark? Note

that we are not discussing the Rosner exotics[9] which are baryon-antibaryon resonances decoupled from the two meson system. We are discussing states like an I = 2 dipion resonance or bound state with a mass near the mass of two pions. If the quarks and antiquarks in two pions attract one another, why is there no net attraction between two positive pions to produce a bound state or a resonance very near threshold?

3. The diquark or meson-baryon puzzle: Why is the quark-quark interaction just enough weaker than the quark-antiquark interaction so that diquarks near the meson mass are not observed, but three-quark systems have masses comparable to those of mesons? Vector gluons which are popular these days would bind the quark-antiquark system, but the force they provide between identical quarks is repulsive. Scalar or other gluons which are even under charge conjugation bind both the quark-antiquark and diquark systems equally. If the quark mass is very heavy, the single quark-antiquark interaction in a meson must cancel two quark masses, while the three quark-quark interactions in the baryon must cancel three quark masses. This suggests that the quark-quark interaction is exactly half the strength of the quark-antiquark interaction.[2] Such a result can be achieved by a suitable mixture of vector and scalar interactions, but it is not very satisfying to obtain such a simple fundamental property of hadrons by a model which fits it with an adjustable parameter.

In all of this discussion, we are considering one-particle states, with the assumption that multiparticle states exist which contain separated particles each having the properties we are trying to explain. Multiparticle states pose additional problems. The allowed spectrum for multiparticle states is not specified by a set of allowed quantum numbers, but by the condition that their constituent particles individually have allowed quantum numbers. Thus the puzzles cannot be answered by general symmetry principles which apply to all states. The triality puzzle is not answered by a symmetry principle forbidding all states which do not have zero triality, because multiparticle states of zero triality must also be forbidden if they are made of particles which individually have nonzero triality. Similarly, the exotics puzzle is not answered by a symmetry principle forbidding all states with exotic quantum numbers because multiparticle exotic states made from nonexotic particles are allowed. Thus any treatment which attempts to answer these puzzles must discuss both single-particle and multiparticle states, and must consider the space-time properties which distinguish between them. Algebraic arguments involving only internal symmetry groups cannot be sufficient.

II. MODELS WITH EXTRA QUARKS

Before considering these puzzles in detail, we review the modified quark models which consider the introduction of additional quark states.

2.1 Color and Charm

In the Gell-Mann-Zweig quark model hadrons are constructed from three quarks which form an SU(3) triplet. Other models with additional quarks have been proposed, primarily for theoretical reasons. Since the GMZ triplet is already sufficient to describe all the known conserved quantum numbers and gives an adequate description of the hadron spectrum (except for a possible difficulty with statistics in the baryons), any additional building block introduces new internal quantum numbers and new degrees of freedom which have not yet been observed, as well as new particle states. All such models require some excuse for throwing these extras away, either by postulating that they are simply not there, or by implying that extra states have a very high mass like the quarks themselves and may be observed at some time in the future.

The additional quarks introduced are of two types, which have been called "charmed" quarks and "colored" quarks. Charmed quarks are simply <u>added</u> to the GMZ triplet to make a total of n quarks and a symmetry SU(n) which includes SU(3) as a subgroup. All the new charmed quarks are singlets in the conventional SU(3). The number of charmed quarks defines a new conserved quantity called charm, analogous to strangeness which is the number of strange quarks. Since no charmed hadrons or hadrons containing charmed quark-antiquark pairs have yet been observed, the charmed quarks are assumed to have a mass sufficiently higher than the conventional triplet so that bound states containing them have a high production threshold.

Colored quarks are sets of n SU(3) triplets, to give a total of 3n quarks. All the new quarks are SU(3) triplets, and the degree of freedom which distinguishes between the different triplets has been recently given the name of "color."

The three-triplet model, originally suggested to allow the three quarks in a baryon to have a symmetric wave function without violating Fermi statistics,[7] is now called a model with "red, white and blue" triplets. For those who find this American chauvinism distasteful, we recommend the "Equal Opportunity Quark Model" (EOQM) which has equal representation of black, white and yellow quarks.

A symmetry group SU(3n) can be defined which treats all quarks on an equal footing. This has a subgroup $SU(3) \times SU(n)_{color}$. There is no evidence for the rich hadron spectrum corresponding to the presence of states classified in nontrivial representations of $SU(n)_{color}$. The observed hadrons are assumed to belong to the trivial singlet representation of SU(n) and "color excitations" of higher representations are either postulated not to exist or are assumed to have a high mass. The color-excited states are sometimes also called charmed states, but there is a definite physical difference between these two types of nonobserved states. The charmed states discussed above contain charmed quarks which are different from those in the observed hadron states. The color-excited states contain exactly the same colored quarks as the observed hadrons, they differ only in having a different permutation symmetry in the space of the colors. Thus charmed states can be pushed up in mass by simply postulating a higher mass for charmed quarks. Color excitations can be pushed up only by having the interaction between quarks depend on the permutation symmetry in color-space since different colored quarks all have the same mass.

In colored quark models, the color may or may not be directly observable. In models where color is not observable, all quarks which differ only in color and otherwise have the same quantum numbers must have the same properties. In other models quarks of different colors have different observable properties, e.g., different electric charges. This possibility has been used to construct models with quarks of integral electric charges. Such integrally-charged colored quarks cannot satisfy the Gell-Mann-Nishijima relation and must have nonzero eigenvalues of a new additive quantum number which appears in the modified Gell-Mann-Nishijima formula. The electromagnetic current then has a component which is an SU(3) singlet and which is not a singlet in $SU(n)_{color}$.

The most recent theoretical models using charmed or colored quarks have been in gauge theories of electromagnetic and weak interactions.[10] These models use symmetries to provide selection rules against undesired weak transitions, such as those due to neutral currents in first or second order. These symmetries are incompatible with SU(3) and a higher symmetry is required to incorporate both SU(3) and the weak interaction symmetry. The higher symmetry requires extra quarks. The suppression of undesired second order transitions is achieved by a cancellation in which unobserved charmed or color-excited intermediate states play a crucial role. These cancellations can occur only if the dominant contribution comes from intermediate states having a much higher energy than the excitation energy of the unobserved states. Thus these models can place upper limits on the excitation energies of these states which can be tested by experiment.

In this talk, we are concerned primarily with the <u>strong</u> interactions which do not depend upon the couplings of quarks to the electromagnetic and weak currents. We therefore do not need to distinguish between models with color-independent fractional charges and models with color-dependent integral charges (e.g., Han-Nambu[11]), since they can have identical strong interactions.

2.2 The Deuteron World

Some insight into the colored quark models is given by the analogy of a world in which all low-lying nuclear states are made of deuterons and have isospin zero; free nucleons have not yet been seen and experiment has not yet attained energies higher than the deuteron binding energy or the symmetry energy required to excite the first $I = 1$ states. In this isoscalar world where all observed states have isospin zero the isovector component of the electromagnetic current would not be observed since it has vanishing matrix elements between isoscalar states. The deuteron energy level spectrum (something like that of a diatomic molecule) would indicate that the deuteron was a two-body system, but there would be no way to distinguish between the neutron and the proton. The deuteron would thus appear to be composed of two identical objects which might be called nucleons. Since the deuteron has electric charge +1, the nucleon would be assumed to have electric charge +1/2. Furthermore, the nucleon would be observed to have spin 1/2 and be expected to satisfy Fermi statistics. However, the ground state of the deuteron and all other observed states would be found to be symmetric in space and spin. Thus, the nucleon would appear to be a spin 1/2 particle with fractional electric charge and peculiar statistics.

Some daring theorists might propose the existence of a hidden degree of freedom expressed by having nucleons of two different colors. There would be a hidden SU(2) symmetry (which might be called isospin) to transform between the two nucleon states of different colors. All the observed low-lying states would be singlets in this new color (or isospin) SU(2). Since the color singlet state of the two-particle system is antisymmetric in the color degree of freedom, the Pauli principle requires the wave function to be symmetric in space and spin, thus solving the statistics problem.

2.3 The Ω^-, With and Without Color

The direct analog of this deuteron problem in hadron quark models is the quark model for the Ω^-. In the conventional quark model, the Ω^- consists of three identical strange quarks (called λ-quarks by some people and s-quarks by others), with their spins of 1/2 coupled symmetrically to spin 3/2. Since the electric charge of the Ω^- is -1, the strange quark is required to have charge -1/3, and it is also required to have peculiar statistics because the system of three identical particles has a symmetric wave function in all known degrees of freedom. Some daring theorists have therefore proposed the existence of a hidden degree of freedom expressed by having strange quarks of three different colors,[7] and a hidden SU(3) symmetry to transform between the three strange quark states of different colors. All the observed low-lying states are singlets in this $SU(3)_{\text{color}}$ group. Since the color-singlet state of the three-particle system is antisymmetric in the color degree of freedom, the Pauli principle requires the wave function to be symmetric in the other degrees of freedom, in agreement with experiment and ordinary Fermi statistics. It is also possible to give these colored strange quarks different integral electric charges, one with charge -1 and two neutrals, by analog with the nucleons in the deuteron. However, as we are concerned primarily with strong interactions, we need not choose between models having different electric charges for colored quarks.

We have chosen the example of the Ω^- for this discussion to simplify the treatment of the conventional SU(3) degree of freedom by considering only strange quarks. When the full triplet of conventional SU(3) is considered, there are three colors for nonstrange as well as for strange quarks, and nine quarks altogether. There are two SU(3) groups, the conventional isospin and hypercharge SU(3) and the color SU(3), which are combined into the direct product $SU(3)_{I,Y} \times SU(3)_{\text{color}}$.

III. THE COLORED GLUON MODEL

We now return to the three puzzles. In the colored quark description of hadrons the restriction that only color singlet states are observed immediately solves the triality puzzle, since only states of zero triality can be color singlets. But requiring all low-lying states to be color singlets is thus equivalent to requiring all low-lying states to have zero triality; it merely replaces one ad hoc assumption with another. What is needed is some dynamical description in which the color singlets turn out to be the low-lying states in a natural way. To attack this problem we return to the fictitious deuteron world where all low-lying states are isoscalar and which is the analog of the colored quark description of hadrons. We follow the treatment of ref. 8.

3.1 A Dynamical Model for the Deuteron World

At first this isoscalar deuteron world seems very artificial. Why should all states with $I = 0$ be pushed down and all states with $I \neq 0$ be pushed up out of sight? But there turns out to be a very natural nuclear interaction which creates exactly this isoscalar deuteron world; namely nuclear two-body forces dominated by a very strong Yukawa interaction provided by ρ exchange. This interaction is attractive for isoscalar states and repulsive for isovector states, <u>in both nucleon-nucleon and nucleon-antinucleon systems</u>. It thus binds only isoscalar states. The ρ-exchange interaction between particles i and j can be expressed in the form

$$v_{ij} = V \vec{t}_i \cdot \vec{t}_j, \tag{1a}$$

where $\vec{t}_i$ is the isospin of particle i and V contains the dependence on all other degrees of freedom except isospin. If we neglect these other degrees of freedom we can write for any n-particle system containing antinucleons and nucleons,

$$V(n) = \frac{1}{2} \sum_{i\neq j} v_{ij} = \frac{1}{2}\left[\sum_{\substack{all\\ij}} v\vec{t}_i \cdot \vec{t}_j - \sum_i V\,\vec{t}_i \cdot \vec{t}_i\right] = \frac{V}{2}[I(I+1) - nt(t+1)] \quad (1b)$$

where I is the total isospin of the system and t is the isospin of one particle; i.e., 1/2 for a nucleon.

The interaction (1b) is seen to be repulsive for the two-body system with I = 1 and attractive for all isoscalar states. A pair of particles bound in the I = 0 state is thus seen to behave like a neutral atom; it does not attract additional particles. Since the pair is "spherically symmetric" in isospace, a third particle brought near the pair sees each of the other particles with random isospin orientation, and its interaction with any member of the pair is described by the average of (1a) over a statistical mixture which is 3/4 isovector and 1/2 isoscalar. This average is exactly zero.

The neutral atom analogy is very appropriate for the description of the observed properties of hadrons. The forces between neutral atoms are not exactly zero, but are much weaker than the forces which bind the atom itself. These interatomic forces produce molecules which are much more weakly bound than atoms. Similarly the forces between hadrons do not vanish but are much weaker than the forces which bind the hadron itself. These interhadronic forces produce complex nuclei which are much more weakly bound than hadrons. In the approximation where we neglect energies much smaller than the quark mass these "molecular" effects are safely neglected.

3.2 The Colored Gluon Interaction for Hadrons

We now generalize this picture for the colored quark description of hadrons. If there are n colors, the interaction (1) must be generalized from SU(2) to SU(n). The quark-antiquark system then still saturates at one pair, but the multiquark system can be seen to saturate at n quarks. A quark-antiquark system which is a singlet in SU(n) exists for all values of n. However, the existence of a singlet in the two-quark system is an accident which occurs only in SU(2) and is not generalizable to SU(n). However the I = 0 two-quark state is also characterized as antisymmetric under permutation of the two particles. This antisymmetry is generalized easily to SU(n) where totally antisymmetric states exist for a maximum of n particles, and the n particle antisymmetric state is a singlet in SU(n).

We now construct the analog of the interaction (1b) for a model with three triplets of different colors. Then the Yukawa interaction produced by the exchange of an octet of "colored gluons" has the form analogous to (1). For an n-particle system containing both quarks and antiquarks,

$$U(n) = \frac{1}{2} \sum_{i \neq j} u_{ij} \sum_{\sigma} g_{i\sigma} g_{j\sigma} \tag{2}$$

where u_{ij} depends on all the noncolor variables of particles i and j and $g_{i\sigma}$ $(\sigma = 1, \ldots, 8)$ denote the eight generators of $SU(3)_{color}$ acting on a single quark or antiquark i.

If the dependence of u_{ij} on the individual particles i and j is neglected, the interaction energy of an n-particle system can be calculated by the same trick used in eq. (1b) to give

$$V(n) = \frac{u}{2}(C - nc) \tag{3a}$$

where u is the expectation value of u_{ij}, integrated over the noncolor variables, C is the eigenvalue of the Casimir operator for $SU(3)_{color}$ for the n-particle system and c = 4/3 is the eigenvalue for a single quark or antiquark. These eigenvalues are directly analogous to the SU(2) Casimir operator eigenvalues $I(I + 1)$ and $t(t + 1)$ in eq. (1b).

In the approximation where all energies small compared to the quark mass M_q are neglected, the interaction (3a) gives the mass formula

$$M(n) = nM_q + V(n) = n(M_q - \frac{cu}{2}) + Cu/2. \tag{3b}$$

The interaction (2) and the mass formula (3b) were first proposed by Nambu,[13] and the saturation properties of the interaction were considered by Greenberg and Zwanziger.[14] However, the remarkable properties of this interaction as demonstrated above in the simplified example of the analogous deuteron world have received little attention.

3.3 Answers to the Triality and Meson-Baryon Puzzles

The formula (3b) can test the triality puzzle or the meson-baryon puzzle by showing whether observable "zero mass" hadron states exist for a given number of quarks and antiquarks. However, it cannot test the exotics puzzle, since it gives no information about the <u>spatial</u> properties of the states. It <u>cannot distinguish between one-particle states and multiparticle scattering states</u> and all zero-triality exotic states are allowed as multiparticle states.

Since C is positive definite and has the eigenvalue zero only for a singlet[8] in $SU(3)_{color}$, and $u \geq 0$ as is evident from the two-body system, the state of the n-particle system with the strongest attractive interaction is a color singlet. Since the interaction is a linear function of n all such singlet states have zero mass if $cu/2 = M_q$. For this case

$$M(n) = (C/c)\,M_q \qquad \text{if } cu/2 = M_q. \tag{3c}$$

The model thus gives observable hadron states for all quark and antiquark configurations for which C = 0 states exist. Since C = 0 states exist only for configurations of triality zero, this answers the triality puzzle.

The meson-baryon puzzle is also answered by this interaction, since zero mass is attained both in two-body and three-body systems. To obtain C = 0, the two-body system must be a quark-antiquark pair, while the three-body system must be a three quark state, totally antisymmetric in color space. The approximation of neglecting the dependence of u_{ij} on i and j is justified in these two cases since there is only one pair in the two-body system, and a totally antisymmetric function has the same wave function for all pairs. The values[8] of the interaction parameter C-nc and the mass parameter C/c are listed in Table I for all states of the two-body system. These show that the quark-quark interaction in the baryon is exactly half of the quark-antiquark interaction in the meson, as required for the meson-baryon puzzle. The diquark mass is thus equal to one quark mass, since its interaction only cancels the mass of one of the two quarks.

TABLE I

Values of the Interaction and Mass Parameters C-nc and C/c

System	$SU(3)_{\text{color}}$	Representation	C	C-nc	C/c
quark-quark	triplet	(antisymmetric)	4/3	-4/3	1
quark-quark	sextet	(symmetric)	10/3	+2/3	5/2
quark-antiquark	singlet		0	-8/3	0
quark-antiquark	octet		3	+1/3	9/4

The interaction averaged over all quark-quark states is seen to be zero and similarly for all quark-antiquark states. An antiquark or quark added to a meson or baryon thus has a zero net interaction, as there can be no color correlations between particles in a singlet state and an external particle, and each pair feels the average interaction over all color states. This suggests that the exotics puzzle is also answered, and that the states of zero mass obtained from the interaction (2) for exotic quantum numbers are multiparticle continuum states rather than bound states or resonances.

3.4 The Exotics Puzzle—Spatial Properties of Wave Functions

To examine the exotics puzzle in more detail we consider the spatial dependence of the interaction (2) for the specific case of the two-quark-two-antiquark system, with an interaction u_{ij} depending only on the positions of the particles and not on momenta, spin and unitary spin. In the representation with the coordinates $\vec{r}_i$ of the four particles diagonal, the interactions u_{ij} are also diagonal and can be treated as c-numbers. In this representation the interaction (2) is a 2×2 matrix in color space as there are two independent couplings for four particles to a color singlet.

We diagonalize this 2×2 matrix to obtain two functions of the coordinates $\vec{r}_i$ which describe the spatial dependence of the interaction in its two color eigenstates.

It is convenient to choose a nonorthogonal basis, related by permutations, which displays quark-antiquark couplings to C = 0,

$$|\alpha\rangle \equiv |(13)_1(24)_1\rangle \tag{4a}$$

$$|\beta\rangle \equiv |(14)_1(23)_1\rangle \tag{4b}$$

where particles 1 and 2 are quarks, 3 and 4 are antiquarks and $(ij)_1$ denotes that particles i and j are coupled to C = 0. Several useful identities follow from the properties of the C = 0 two-particle state.

$$\langle\alpha|\beta\rangle = 1/3 \tag{5a}$$

$$\sum_\sigma g_{1\sigma}g_{3\sigma}|\alpha\rangle = \sum_\sigma g_{2\sigma}g_{4\sigma}|\alpha\rangle = -(8/3)|\alpha\rangle \tag{5b}$$

$$\sum_\sigma g_{1\sigma}g_{4\sigma}|\beta\rangle = \sum_\sigma g_{2\sigma}g_{3\sigma}|\beta\rangle = -(8/3)|\beta\rangle \tag{5c}$$

$$(g_{1\sigma}+g_{3\sigma})|\alpha\rangle = (g_{2\sigma}+g_{4\sigma})|\alpha\rangle = (g_{1\sigma}+g_{4\sigma})|\beta\rangle = (g_{2\sigma}+g_{3\sigma})|\beta\rangle = 0 \tag{5d}$$

$$\langle\alpha|g_{1\sigma}g_{4\sigma}|\alpha\rangle = \langle\beta|g_{1\sigma}g_{3\sigma}|\beta\rangle = 0 \tag{5e}$$

$$\langle\alpha|g_{1\sigma}g_{4\sigma}|\beta\rangle = \langle\beta|g_{1\sigma}g_{3\sigma}|\alpha\rangle = -(8/3)\langle\alpha|\beta\rangle = -8/9. \tag{5f}$$

By operating with the interaction (2) on the wave functions (4) and eliminating the color variables with the aid of the identities (5) we obtain

$$-3U|\alpha\rangle = (8u_\alpha - u_\beta + u_q)|\alpha\rangle + 3(u_\beta - u_q)|\beta\rangle \tag{6a}$$

and

$$-3U|\beta\rangle = 3(u_\alpha - u_q)|\alpha\rangle + (8u_\beta - u_\alpha + u_q)|\beta\rangle \tag{6b}$$

where

$$u_\alpha = u_{13} + u_{24}; \quad u_\beta = u_{14} + u_{23}; \quad u_q = u_{12} + u_{34}. \tag{7}$$

Solving the secular equation for eqs. (6) gives the eigenvalues for U,

$$U' = -(7/6)(u_\alpha + u_\beta) - (1/3)u_q \pm (1/2)\sqrt{8(u_\alpha - u_\beta)^2 + (u_\alpha + u_\beta - 2u_q)^2}. \qquad (8)$$

If u_{ij} is a finite range potential which vanishes at large distances, the eigenvalues (8) reduce to those for two independent two-particle clusters for all values of the coordinates $\vec{r}_i$ which correspond to two pairs separated by a distance greater than the range of the potential. The case $u_\beta = u_q = 0$ describes such a separation between the pairs of particles (13) and (24). The corresponding eigenvalues from eq. (8) are $U' = -(8/3)u_\alpha$ and $U' = +(1/3)u_\alpha$ exactly those of Table I for two separated quark-antiquark pairs in the singlet and octet states. The case $u_\alpha = u_\beta = 0$ describes separated pairs of like particles (12) and (34) and has eigenvalues $U' = -(3/4)u_q$ and $U' = +(2/3)u_q$ exactly those of Table I for two separated quark-quark and antiquark-antiquark systems in the triplet and sextet states.

To test the exotics puzzle we look for coordinate configurations where four-particle correlations may give stronger binding than in two noninteracting clusters. Since u_α and u_β appear symmetrically in (8), we need only consider values of $u_\beta \leq u_\alpha$. For any value of u_α the value of $u_\beta \leq u_\alpha$ which minimizes the interaction (8) is $u_\beta = u_\alpha$ with the negative sign for the square root. This gives

$$U' = -(8/3)u_\alpha - (2/3)(u_\alpha - u_q). \qquad (9)$$

This expression is minimized by choosing the minimum value of u_q consistent with a given value of u_α. For monotonically decreasing potentials this is achieved by placing the four particles at the corners of a square with the like particles at opposite diagonals.

For a square well potential the particles can be arranged in a square with the diagonal greater than the range of the forces and the sides less than the range. This configuration has $u_q = 0$ and forms a stable four-particle state with a binding 25% greater than that of two quark-antiquark pairs. However, the sharp edge of the square well is essential for this binding and does not seem reasonable physically. For smooth potentials without sharp edges such as Gaussian, Yukawa or harmonic oscillator potentials eq. (9) shows that such a four-particle cluster is less strongly bound than two noninteracting quark-antiquark pairs, and the system simply breaks up into two clusters. This leads to a description in which all states having exotic quantum numbers are just scattering states of particles which individually have nonexotic quantum numbers, and answers the exotics puzzle.

IV. CONCLUSIONS

An important conclusion from this discussion, irrespective of the validity of the colored gluon model, is that of the crucial role played by repulsive forces and the extra color degree of freedom in finding answers to the three puzzles. The conventional quark model has observed mesons <u>in all states that can be constructed from a quark-antiquark pair</u>. This has been hailed as a great success of the quark model. But perhaps this is not a success but a failure. With bound states in all channels <u>the quark-antiquark force is required to be attractive in all channels</u> and cannot prevent an antiquark from being bound to a baryon. With dominant two-body forces and no adhoc three-body forces, antiquark-baryon binding will occur unless the quark-antiquark interaction is repulsive in many states and repulsion dominates the antiquark-baryon interaction for all possible states of the antiquark. These many repulsive channels are provided in the three-triplet model which has 81 $q\bar{q}$ states where only nine are observed. The absence of the other 72 "charmed" states should not be held against the model, as repulsive quark-antiquark forces in all charmed channels not only explain this absence but also provide the necessary repulsion to explain the absence of all $3q\bar{q}$ bound states.

In the conventional quark model, vector gluons cannot bind both mesons and baryons, because the vector interaction between identical particles is repulsive and cannot bind states like the Ω^-, Δ^{++} and Δ^-

composed of three identical quarks. In the three-triplet model, the three quarks in each of these hadrons are no longer identical while all states containing three identical quarks are "charmed states" and should have a high mass. Thus repulsion between identical quarks is actually desired in the three-triplet model and there is no objection to vector gluons.

In the three-triplet model "color" correlations absent in the conventional quark model increase binding and help in saturation. The quantum numbers of individual bound quarks are not fixed; they change in a correlated way as a result of exchange forces. With Nambu's interaction[13] the two-body forces are attractive in certain color correlated states, while the interaction in an uncorrelated state (averaged over color) is zero, as repulsions and attractions cancel exactly. A color singlet state thus behaves like a neutral atom. There are no color correlations between an external particle and a quark in a color singlet and the interactions cancel.

The three-triplet model with colored vector gluon binding gives a good description of those dominant features of the hadron spectrum expected in zero order: namely that only the quark-antiquark and three-quark systems appear as bound states with a mass of approximately zero on the scale of the quark mass. This is in contrast with other models whose zero order results are in violent disagreement with experiment. These are unable to explain the three puzzles discussed in this paper, and can only hope that a better theory makes the puzzles go away. Further refinements in the description of the hadron spectrum can be expected in the framework of the three-triplet vector-gluon model by considering higher order effects, such as the additional phenomenological potentials commonly used[15] to calculate the hadron spectrum. These are entirely consistent with the three-triplet model since the symmetric quark model[7] is commonly used in all treatments for baryons. The three-triplet model should therefore be considered very seriously as an alternative to the single-quark model.

APPENDIX

Troubles with Parton Models for Inclusive Meson Production

This point was raised during the talk and is added here.

It is very tempting to make models for inclusive meson production in which a quark finds an antiquark partner and the two escape as a meson.[16] But the spins of the quarks cannot simply be ignored. If there are no spin correlations between the quark and antiquark, a quark-antiquark pair has a 75% probability of being in the triplet spin state and only a 25% probability of being in the singlet spin state. The triplet state is a meson resonance, which then decays to produce two or more pseudoscalar mesons. The singlet state can either be a pseudoscalar meson or a higher meson resonance. Thus at least three meson resonances are produced for every directly produced pseudoscalar meson and pseudoscalar meson production via resonance decay is at least six times greater than direct production. Kinematic factors may discriminate against resonance contributions because of the higher masses of resonances and because the resonance decay products individually have lower momenta than a directly produced meson. But it is clearly very risky to overlook resonance contributions. Any model which considers only direct production of pseudoscalar mesons must be viewed with suspicion.

As an example, consider the production of kaons and pions both directly and via decays of the vector nonet and the η. There is no obvious reason for leaving out tensor and higher mesons, but their presence can only make matters worse. Let $\hat{\sigma}(M)$ denote the cross section for the direct production of meson M and $\sigma(M)$ denote the total cross section which includes production via resonance decays. Then

$$\sigma(K^+) = \hat{\sigma}(K^+) + \frac{1}{3}\hat{\sigma}(K^{*+}) + \frac{2}{3}\hat{\sigma}(K^{*0}) + \frac{1}{2}\hat{\sigma}(\phi) \tag{A1a}$$

$$\sigma(K^0) = \hat{\sigma}(K^0) + \frac{2}{3}\hat{\sigma}(K^{*+}) + \frac{1}{3}\hat{\sigma}(K^{*0}) + \frac{1}{2}\hat{\sigma}(\phi) \tag{A1b}$$

$$\sigma(K^-) = \hat{\sigma}(K^-) + \frac{1}{3}\hat{\sigma}(K^{*-}) + \frac{2}{3}\sigma(\overline{K}^{*0}) + \frac{1}{2}\hat{\sigma}(\phi) \tag{A1c}$$

$$\sigma(\overline{K}^0) = \hat{\sigma}(\overline{K}^0) + \frac{2}{3}\hat{\sigma}(K^{*-}) + \frac{1}{3}\sigma(\overline{K}^{*0}) + \frac{1}{2}\hat{\sigma}(\phi) \tag{A1d}$$

$$\sigma(\pi^+) = \hat{\sigma}(\pi^+) + \hat{\sigma}(\rho^+) + \hat{\sigma}(\rho^0) + \hat{\sigma}(\omega) + x\hat{\sigma}(\eta) + \frac{2}{3}\hat{\sigma}(K^{*+}) + \frac{2}{3}\hat{\sigma}(\overline{K}^{*0}) \tag{A2a}$$

$$\sigma(\pi^-) = \hat{\sigma}(\pi^-) + \hat{\sigma}(\rho^-) + \hat{\sigma}(\rho^0) + \hat{\sigma}(\omega) + x\hat{\sigma}(\eta) + \frac{2}{3}\hat{\sigma}(K^{*-}) + \frac{2}{3}\hat{\sigma}(K^0) \tag{A2b}$$

$$\sigma(\pi^0) = \hat{\sigma}(\pi^0) + \hat{\sigma}(\rho^+) + \hat{\sigma}(\rho^-) + \hat{\sigma}(\omega) + x\hat{\sigma}(\eta) + \frac{1}{3}\hat{\sigma}(K^{*+}) + \frac{1}{3}\hat{\sigma}(K^{*-})$$

$$+ \frac{1}{3}\hat{\sigma}(K^{*0}) + \frac{1}{3}\hat{\sigma}(\overline{K}^{*0}) \tag{A2c}$$

where the factors 1/3, 2/3 and 1/2 are isospin Clebsch-Gordan coefficients which give branching ratios of different charged modes (isospin breaking is neglected) and x denotes the fraction of $\eta \to \pi^+\pi^-\pi^0$ decays. Other η decays producing pions are neglected.

Let us now assume that vector mesons are produced three times more frequently than pseudoscalars because of the statistical spin factor. Then

$$\hat{\sigma}(K^*) = 3\hat{\sigma}(K) \tag{A3a}$$

$$\hat{\sigma}(\rho) = 3\hat{\sigma}(\pi) \tag{A3b}$$

$$\hat{\sigma}(\omega) = 3\hat{\sigma}(\pi^0), \tag{A3c}$$

where eqs. (A3a) and (A3b) hold individually for any charge state. We also assume that η production is less than π^0 production by some factor and write

$$x\hat{\sigma}(\eta) = y\hat{\sigma}(\pi^0), \tag{A3d}$$

where $y < 1$.

Then from eqs. (A3), (A1) and (A2), we obtain

$$\sigma(K^+) = \sigma(K^0) = 2[\hat{\sigma}(K^+) + \hat{\sigma}(K^0)] + \frac{1}{2}\hat{\sigma}(\phi) \tag{A4a}$$

$$\sigma(K^-) = \sigma(\overline{K}^0) = 2[\hat{\sigma}(K^-) + \hat{\sigma}(\overline{K}^0)] + \frac{1}{2}\hat{\sigma}(\phi) \tag{A4b}$$

$$\sigma(\pi^+) = 4\hat{\sigma}(\pi^+) + (6+y)\,\hat{\sigma}(\pi^0) + 2\hat{\sigma}(K^+) + 2\hat{\sigma}(\overline{K}^0) \quad \text{(A5a)}$$

$$\sigma(\pi^-) = 4\hat{\sigma}(\pi^-) + (6+y)\,\hat{\sigma}(\pi^0) + 2\hat{\sigma}(K^-) + 2\hat{\sigma}(K^0) \quad \text{(A5b)}$$

$$\sigma(\pi^0) = (4+y)\,\hat{\sigma}(\pi^0) + 3\hat{\sigma}(\pi^+) + 3\hat{\sigma}(\pi^-) + \hat{\sigma}(K^+) + \hat{\sigma}(K^-) + \hat{\sigma}(K^0) + \hat{\sigma}(\overline{K}^0). \quad \text{(A5c)}$$

Equations (A4) give the surprising result that $\sigma(K)$ and $\sigma(\overline{K})$ production rates are independent of the charge of the kaon, even though the direct production rate may be charge dependent; e.g., $\sigma(K^+) = \sigma(K^0)$ even when $\hat{\sigma}(K^+) \neq \hat{\sigma}(K^0)$. This results because the contributions from K^* decays and direct production have opposite charge asymmetry. The pion has a higher isospin than a kaon; thus the charge asymmetry in the K^* distribution shows up in the decay pions, and the charge asymmetry in the decay kaons is the reverse. The exact cancellation of the charge asymmetries in direct and resonance productions results from the values of the Clebsch-Gordan coefficients.

Other quantities of interest which are obtained from eqs. (A4) and (A5) are the total pion and kaon production cross sections summed over all charge states. We denote these by $\sigma_{tot}(M)$.

$$\sigma_{tot}(K) = 4\hat{\sigma}_{tot}(K) + 2\hat{\sigma}(\phi) \quad \text{(A6a)}$$

$$\sigma_{tot}(\pi) = 7\hat{\sigma}_{tot}(\pi) + (9+3y)\,\hat{\sigma}(\pi^0) + 3\hat{\sigma}_{tot}(K) \quad \text{(A6b)}$$

$$= (10+y)\,\hat{\sigma}_{tot}(\pi) + (3+y)[2\hat{\sigma}(\pi^0) - \hat{\sigma}(\pi^+) - \hat{\sigma}(\pi^-)] + 3\hat{\sigma}_{tot}(K). \quad \text{(A6c)}$$

The ratio of pion to kaon production is seen to be considerably larger than predictions from direct production. For example, if we neglect the small η and ϕ contributions and assume equal direct production cross sections for all pion and kaon states, the π/K ratio is enhanced by a factor of 7/2 over the value predicted by direct production.

Another interesting relation is

$$\sigma(\pi^+) + \sigma(\pi^-) - 2\sigma(\pi^0) = -2[\hat{\sigma}(\pi^+) + \hat{\sigma}(\pi^-) - 2\hat{\sigma}(\pi^0)]. \quad \text{(A7)}$$

This particular linear combination of pion cross sections is predicted to vanish by the quark parton model. It also vanishes in any model which produces the pions via fragmentation[17] of an object with isospin 1/2. It is thus not surprising that the quantity (A7) vanishes for the total cross sections if it vanishes for the direct production cross sections, since the relevant resonance production and decay conserves isospin. ($\eta \to \pi^+\pi^-\pi^0$ decays do not contribute to this quantity.) However, it is amusing that any deviation from zero in the direct production has its sign reversed in the total production. If the direct production of neutral pions is <u>less</u> than half the direct production of charged pions, the total production of neutral pions will be <u>greater</u> than half the total production of charged pions.

1. M. Gell-Mann, Acta Physica Austriaca, Suppl. IX, 733-761 (1972).
2. H. J. Lipkin, Quarks for Pedestrians, Weizmann Institute preprint WIS-73/8-Ph, to be published in Physics Reports.
3. F. J. Gilman and M. Kugler, Phys. Rev. Letters 30, 518 (1973); H. J. Melosh IV, Caltech Thesis (1973 unpublished. M. Kugler, these proceedings.
4. S. Yoshida and L. Zamick, Annual Review of Nuclear Science 22, 121 (1972). See in particular Tables 4 and 5 and section VIII.
5. G. E. Brown, Unified Theory of Nuclear Models, North-Holland Publishing Company, Amsterdam (1967), Chapter VIII.
6. A. Katz, Journ. of Math. Phys. 10, 2215 (1969).
7. O. W. Greenberg, Phys. Rev. Letters 13, 122 (1964); O. W. Greenberg and M. Resnikoff, Phys. Rev. 163, 1844 (1967).
8. H. J. Lipkin, Phys. Letters 45B, 267 (1973).
9. J. L. Rosner, Phys. Rev. Letters 21, 950 (1968); P. G. O. Freund, R. Waltz and J. Rosner, Nucl. Phys. B13, 237 (1969).
10. J. Primack, these proceedings. J. D. Bjorken, in Proceedings of the XVI International Conference on High Energy Physics, Edited by J. S. Jackson and A. Roberts, National Accelerator Laboratory (1972), Vol. 2, p. 299;

    H. J. Lipkin, Phys. Rev. D7, 1850 (1973).
11. M. Y. Han and Y. Nambu, Phys. Rev. 139, B1006 (1965).
12. H. J. Lipkin, Symmetries and Resonances (Theoretical), in Particle Physics (Irvine Conference, 1971), edited by M. Bander, G. L. Shaw and D. Y. Wong (American Institute of Physics, New York, 1972), p. 30.
13. Y. Nambu, in Preludes in Theoretical Physics, edited by A. de-Shalit, H. Feshbach and L. Van Hove (North-Holland Publishing Company, Amsterdam (1966), p. 133.
14. O. W. Greenberg and D. Zwanziger, Phys. Rev. 150, 1177 (1966).

15. R. H. Dalitz, in Quarks and Symmetries, Proceedings of International Conference on Symmetries and Quark Models, Detroit 1969, Edited by Ramesh Chand, Gordon & Breach, New York, 1970, p. 355, and in Methods and Problems of Theoretical Physics, Edited by J. E. Bowcock, North-Holland Publishing Co., Amsterdam (1970), p. 343.

16. G. Feldman, these proceedings.

17. H. J. Lipkin and M. Peshkin, Phys. Rev. Letters 28, 862 (1972).